TWO-DIMENSIONAL FLOW OF A NON-NEWTONIAN FLUID IN THE CHANNEL OF SCREW MACHINERY WITH WALL SLIP

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The isothermal flow of an anomalously viscous fluid in a screw channel is investigated with allowance for circulation flow of the fluid and the side ribs of the channel.

Non-Newtonian fluid flow obeying a power-law rheological equation in the channel of screw machinery (extruders, pumps, etc.) has been investigated under simple shear conditions with slip [1, 2] and under complex shear conditions [3, 4]. In these papers the friction of the fluid at the side ribs is disregarded, i.e., it is assumed that the velocity components and the effective viscosity of the fluid are invariant over the width of the channel and the radial velocity is equal to zero. Since the side walls of the channel can significantly affect the flow pattern in screw machinery [5, 6], it is important to investigate this problem in a two-dimensional setting.

For the solution of the problem we assume that clearances between the ridges of the screw and the casing (Fig. 1a) are absent and that the depth of the screw channel H is much smaller than the radius of the casing. We can therefore work with a plane model of the screw channel (Fig. 1b), and to facilitate the analysis of the fluid flow pattern we invert the motion, i.e., regard the screw as stationary and the casing as rotating with a constant linear velocity V_0. We consider the fluid flow to be steady, laminar, and isothermal. The motion of the material in the screw channel in this case is described by a system of differential equations including the equations of motion and continuity as well as the constitutive equations. This system must be closed by suitable boundary conditions. In Cartesian coordinates the indicated equations take the form

$$\rho v_j \frac{\partial v_i}{\partial x_j} = \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}, \tag{1}$$

$$\frac{\partial v_i}{\partial x_i} = 0, \tag{2}$$

$$\tau_{ij} = \eta \left(\frac{I_2}{2}\right) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{3}$$

The solution of the system (1)-(3) in the general case poses a difficult task even with the application of modern computer techniques and equipment. However, it can be enormously simplified by assuming that the velocities of the fluid are invariant along the length of the channel (z axis), i.e., that all derivatives of the velocities with respect to the coordinate z are equal to zero.

Before undertaking the solution of the problem, we formulate the slip boundary conditions. In transferring the results of viscosimetric studies (simple shear) to a complex stress state we invoke the "common curve" hypothesis, i.e., we assume that in both simple and complex flow the slip velocity v_W is described by the same equation [3]

$$v_W = \frac{\beta_W(\tau_W)\, \tau_W}{H^m}. \tag{4}$$

In the complex stress state, however, the coefficient β_W no longer depends on the shear stress τ_W, but on the tangential stress intensity T_W at the wall.

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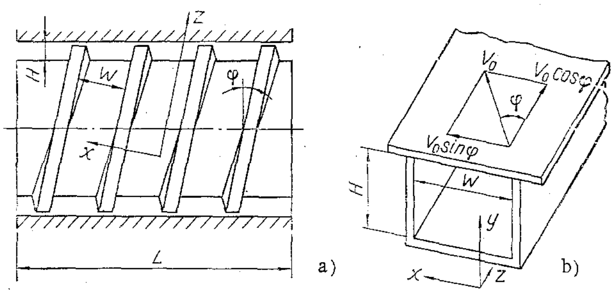

Fig. 1. Schematic representation of the screw (a) and its plane model (b).

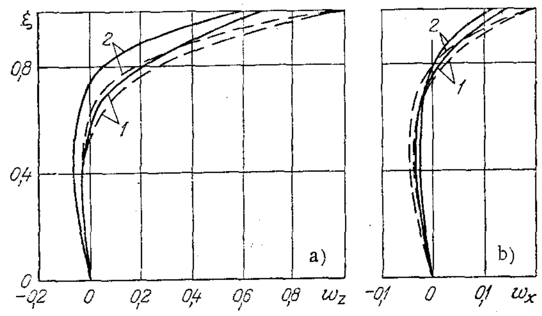

Fig. 2. Profiles of the dimensionless fluid velocities w_z (a) and w_x (b) at $x = W/2$. 1) Two-dimensional flow; 2) complex shear model.

In light of the foregoing remarks the boundary conditions for the velocities v_z and v_x at, for example, $y = H$ are written as follows:

$$v_z = V_0 \cos\varphi + \frac{\beta_w(T_w)\,\tau_{yz}}{H^m},$$
$$v_x = V_0 \sin\varphi + \frac{\beta_w(T_w)\,\tau_{xy}}{H^m}. \tag{5}$$

The boundary conditions for the velocities with slip at the other walls are formed analogously.

To solve the system of equations (1)-(3) the method of finite differences is used extensively in fluid mechanics. As a rule, this system of equations is not written in terms of the velocities and pressure, but in terms of new independent variables: the vorticity ω, stream function ψ, and velocity v_z. When the slip effect is present in the fluid, the primary task is the derivation of boundary conditions for the vorticity ω, because the boundary conditions for the velocity v_z are given by expressions of the type (5) and those for ψ as in the no-slip situation.

In deriving the finite-difference formula for the vorticity at the boundary we expand the stream function ψ into a Taylor series in powers of its increment in the direction perpendicular to the solid wall and we make use of the familiar relation between ω and ψ. Then in the most general case [bearing in mind that $v_w = v_w(\tau_w, H)$], with four terms retained in the Taylor series, the following equation is obtained:

$$\omega_1 = -\left[\frac{3(\psi_2 - \psi_1)}{\rho h^2} + \frac{\omega^2}{2}\right] + \frac{3v_w}{h}. \tag{6}$$

In this equation v_w must be interpreted as the velocity calculated according to expressions of the type (5), for example, v_x. The subscripts 1 and 2 refer to the nodes of the computing grid at the wall and at a distance h from it equal to the grid step.

If, on the other hand, Eq. (4) is taken as the law governing the slip velocity, then the expression for the vorticity at the boundary is obtained in the form

$$\omega_1 = -\left[\frac{3(\psi_2 - \psi_1)}{\rho h^2(1 + 3\beta\eta/h)} + \frac{\omega_2}{2(1 + 3\beta\eta/h)}\right] + \frac{3V_0}{h(1 + 3\beta\eta/h)}, \tag{7}$$

where $\beta = \beta_w(T_w)/H^m$.

We have solved the stated problem on a BÉSM-6 computer, using a FORTRAN program. For the specific calculations we use a fluid whose velocity obeys the power-law flow equation

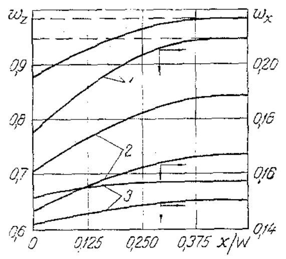

Fig. 3

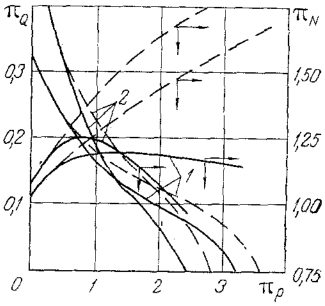

Fig. 4

Fig. 3. Variation of the dimensionless velocities w_z and w_x over the width of the screw channel at the moving boundary for various values of the dimensionless pressure gradient under slip (solid curves) and no-slip (dashed curves) conditions. 1) $\pi_p = 0.293$; 2) 1.485; 3) 2.93.

Fig. 4. Dimensionless values of the volumetric flow rate π_Q and power π_N versus dimensionless pressure gradient π_p. The nomenclature is the same as in Fig. 2.

$$\eta = \eta_0 \left(\frac{I_2}{2}\right)^{\frac{n-1}{2}}, \tag{8}$$

while the slip ratio $\beta_w(T_w)$ obeys the law

$$\beta_w(T_w) = \begin{cases} 0 & \text{for } T_w < \tau_s, \\ b(T_w - \tau_w) & \text{for } T_w > \tau_s. \end{cases} \tag{9}$$

The other parameters have the values H = 0.005 m; W = 0.02 m; $\varphi = 12°$; $V_0 = 0.1$ m/sec; n = 0.27; $\eta_0 = 1.52 \cdot 10^5$ Pa·sec^n; m = 2.5; $\tau_s = 4.5 \cdot 10^5$ Pa; b = $1.529 \cdot 10^{-18}$ m^{m+5}/N^{2}sec.

Figure 2 gives the results of the calculations in the form of the dimensionless velocities $w_z = v_z/V_0$ (Fig. 2a) and $w_x = v_x/V_0$ (Fig. 2b) at x = 0.01 m (in the central part of the channel) and the values of the dimensionless pressure gradient $\pi_p = A_z\overline{H}/\overline{\tau} = 2.344$. The solid curves correspond to fluid flow with slip, and the dashed curves to no-slip flow. Curves 2 in the same figure represent data for the complex shear model of [3] (without regard for fluid friction at the side ribs of the channel).

Figure 3 shows the distributions of the dimensionless velocities w_z and w_x of the fluid at the upper boundary $\xi = y/H = 1$ along the width of the screw channel as a function of the dimensionless pressure gradient π_p. The influence of the side walls is tantamount to the initiation of slip (for small gradients π_p) in the upper corners of the channel and then, as π_p is increased, gradual entrainment of the entire surface. It is evident from the figure that the maximum slip velocity is observed in the regions contiguous with the side faces of the channel, and the minimum in the central part of the channel.

Figure 4 shows the dimensionless values of the volumetric flow rate of the fluid $\pi_Q = Q/V_0HW$ and the power consumption $\pi_N = N/V_0W\overline{\tau}$ as a function of the dimensionless pressure gradient π_p. It is evident from the figure that the complex shear model gives too large a value for the fluid flow rate and too small a value for the power in both the slip-flow and the no-slip situation.

The most interesting results, in our opinion, are afforded by the $\pi_N - \pi_p$ curves (Fig. 4). The power consumed by the screw mechanism is calculated according to the equation

$$N = \int_0^l \int_0^W \int_0^H \eta\left(\frac{I_2}{2}\right) \frac{I_2}{2}\, dxdydz + \int_{p_0}^{p_1} Qdp \quad \left(l = \frac{L}{\sin\varphi}\right). \tag{10}$$

It is apparent from Fig. 4 that initially, as long as fluid slip is absent, the power consumed by the screw mechanism increases with the pressure gradient (i.e., with decreasing fluid flow rate), but then, starting with a certain gradient π_p, fluid slip is observed, and the power continues to grow with the pressure at first, and then begins to decrease. It is interesting to note that the drop in power with increasing value of π_p proceeds far more slowly for the two-dimensional model than for the complex shear model.

It is evident from an analysis of the foregoing results that, first, wall slip exerts a powerful influence on the performance characteristics of the mechanism and, second, the effect of the side walls in conjunction with fluid slip along the walls is very pronounced and must be taken into consideration in the calculations.

NOTATION

m, n, η_0, b, rheological constants; H, W, depth and width of screw channel; V_0, circumferential velocity of screw ridges (velocity of upper plate in the opposite direction); φ, angle of elevation of screw line; ρ, fluid density; x, y, z, x_i, x_j, Cartesian coordinates; L, screw length; l, length of screw channel; i, j = 1, 2, 3; w_x, x_z, dimensionless fluid velocities; v_x, v_z, v_i, v_j, true fluid particle velocities; v_w, wall slip velocity; ω, vorticity; ψ, stream function; I_2, second (quadratic) invariant of strain-rate tensor; η, effective fluid viscosity; T_w, tangential stress intensity at wall; h, step of computing grid; Q, volumetric flow rate; N, power; A_z, pressure gradient along screw channel; π_Q, π_p, π_N, dimensionless values of flow rate, longitudinal pressure gradient, and power; $\xi = y/H$, dimensionless coordinate; p, pressure; p_0, p_1, fluid pressures at channel entrance and exit; $\bar{\tau} = \eta_0(V_0/H)^n$, characteristic tangential stress; τ_{ij}, components of stress tensor; τ_s, shear stress at which slip is initiated; τ_w, shear stress at wall; β_w, slip ratio.

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THERMAL CONDITIONS OF MAGNETOFLUID SEALS

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Heat production in the working region of magnetofluid seals is theoretically and experimentally evaluated.

One of the most promising sealing techniques for application to rapidly rotating systems is the magnetofluid seal (MFS). The MFS, whose working element is a ferromagnetic fluid (FMF), held in a prescribed position by a magnetic field, has several advantages over the common contact and noncontact seals: MFS operate is a wide range of shaft rotation speeds, have a low friction torque and a long operating life [1].

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